N RESPONSE FUNCTIONS FOR TODAY'S PRODUCTION COSTS

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ABSTRACT

Building upon a discussion of linear and curvilinear yield response to fertilizer, this research develops a framework where response is fundamentally linear for any particular site-year, but where expected response can become curvilinear in the face of random weather across space and time. We next put forth several functional forms as potential candidates for generalizing expected yield response to N. Functional forms were evaluated using historical N trial data from western and north central Kansas involving wheat, corn, and grain sorghum. The quadratic plateau arose as the functional form of choice from that analysis. Given the doubling of fertilizer N prices in the last two years, that functional form implies N cutbacks around 10 lb/acre for 50 bu/acre wheat to around 30 lb/acre for 250 bu/acre corn. Finally, we present the mathematics required to compute our suggested adjustments to current KSU N recommendations to accommodate changes in N prices.

BACKGROUND

Nitrogen fertilizer (*fertN*) prices have increased markedly over the last few years. Moreover, grain prices for N-using crops such as wheat and corn have not kept pace with fertilizer prices. Naturally, farmers are asking questions like, Should I back off my fertilizer rates? If so, how much? Should I change my application practices to get a bigger bang for my fertilizer buck? If so, how much can I reduce rates when I do this? Such questions can only be answered through an understanding of crop yield response to fertilizer. Hence, this paper develops reasonable yield response functions (yield models) for Kansas wheat, corn, and grain sorghum – functions that result in suggested adjustments to Kansas State University's (KSU) N recommendations (*Nrec*) for consideration by crop decision makers in the face of rapidly changing N prices.

A mathematical yield response function generally is depicted as a two dimension graph, with crop yield on the y-axis and fertilizer rate on the x-axis. As a first cut, such yield response functions can be separated between those showing a constant yield increase for each successive lb/acre of fertilizer, at least up to a point, and those that depict a diminished response with each successive unit of fertilizer. Constant, or linear, response often is represented as a linear plateau, where the yield-responsive part of the underlying graph is a straight upward-sloping line that becomes horizontal at some sufficiently high fertilizer rate. The idea is that each additional unit of fertilizer induces a constant yield increase, but at some point, typically due to some other crop production factor becoming limiting, yield will no longer respond to increases in fertilizer. On the other hand, non-linear response is depicted as a curve that starts out steep when fertilizer rates are low, then gradually flattens out with increased fertilizer rates. This is referred to as diminishing returns to fertilizer. Sometimes, as with the linear plateau, non-linear response is "chopped off" by a horizontal line at some level of fertilizer, once again supportive of the idea that other factors become limiting.

The shape of the yield response function has significant economic implications. The slope of the line at any point depicts the increased yield expected from adding another unit of fertilizer. For example, consider an arbitrary slope of 0.2, which implies that the next lb/acre of fertilizer is expected to induce a yield bump of 0.2 bu/acre. If grain is valued at $2.00/bu, that increase in yield will be worth $0.40/acre. As long as fertilizer costs less than $0.40/lb, it will pay to apply that next lb/acre. With non-linear response, the slope of the line diminishes with increased fertilizer. Thus, at some point, the increased yield revenue from the next lb/acre of fertilizer will no longer cover its cost and that becomes the economic optimum (profit-maximizing) level of fertilizer to apply. On the other hand, with a linear response, the slope of the line is a constant. If that constant is above the fertilizer-to-crop price ratio, and ignoring application costs, then the last (just before the plateau) lb/acre of fertilizer will pay just as well as the first lb/acre. In short, the economic optimum fertilizer rate always will be either 0 or that level resulting in the plateau yield, never anything in between. Clearly, with a linear plateau, the optimum fertilizer rate will be the same for a wide range of fertilizer and crop prices. Only at extremely high fertilizer prices will the optimum fertilizer rate change, whereupon it drops to 0. But, with a non-linear response function, a change in fertilizer or crop price will induce a change in the economic optimum fertilizer rate.

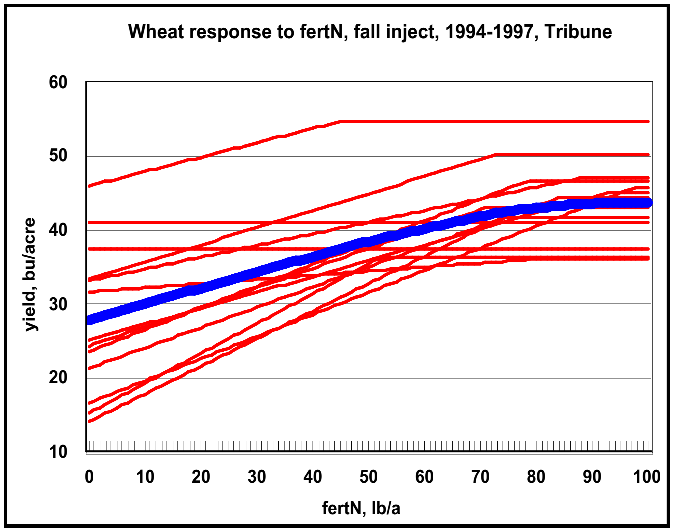

Figure 1

So, considering specifically N, is yield response linear or curvilinear? The academic literature is mixed on this. Research reported by economists tends to depict yield response to N as curvilinear. But, agronomy, especially in its fundamental textbooks, often depicts a linear response. Perhaps the best answer to the question is that yield response to N may be both linear and curvilinear. That is, response is *fundamentally* linear, meaning that yield response to N in any particular site-year of a fertilizer rate research trial is best depicted as a linear plateau. But, considerable random variation in other limiting factors across space and time, especially weather, causes *expected* response to be either linear or curvilinear, a purely empirical issue. For example, Fig. 1 shows a series of linear plateaus, each generalizing (fitting with least squared errors) the yield and N rate information from a unique site-year in an N study for wheat conducted by Alan Schlegel of KSU's Southwest Research-Extension Center in Tribune, Kansas. Assuming the variations among the linear plateaus in the figure are largely weather induced, a farmer likely would consider the average of such values above a particular *fertN* rate to be his best guess of the yield he will get when applying that rate. The heavy line in the figure depicts these averages at the various measured or interpolated *fertN* levels. Practically, this collection of short connecting line segments, which appears slightly curvilinear, implies that incremental changes in *fertN* price will lead to incremental changes in optimal *fertN* rates, as described earlier in the discussion of non-linear response.

There are good reasons to treat the collection of line segments comprising the heavy line in Fig. 1 as a continuous mathematical function (a yield model). First, it is likely that a smoothly curved line would have emerged had more and more site-years been included in the study. Second, a well-defined mathematical function is easier to work with than a collection of line segments when it comes to making decisions from the information generalized. That said, the nature of the mathematical function can be important. For example, in plateau-type functions, it is the degree of non-linearity (the sharpness of the yield response curve) close to the yield plateau that determines how much fertilizer and/or crop prices impact optimum fertilizer rates.

Though not universal, most mathematical formulas used by universities and soil testing laboratories for generating fertilizer rate recommendations do not have a price component. Likely, the fact that price is absent from these formulas indicates an implicit yield response function that curves sharply (flattens rapidly) near the top. That is, even though price would matter to such yield models, it may not matter much. Put another way, higher fertilizer prices would induce only slightly lower optimal fertilizer rates and hence similar yields. In short, if price matters little to optimal rates, then it may not be worth the cost of making explicit such price impacts on fertilizer rates. Also, ignoring price will not be particularly costly for a decision maker using such recommendations.

In some sense, the tradition of ignoring price impacts on optimal fertilizer rates via explicit yield response functions is still being practiced today. For example, based on informal communications through internet message boards and in person, it appears that many crop consultants and educators currently are focusing their educational response to high fertilizer prices on ad hoc yield goal reductions or in areas of application efficiency rather than in areas of yield response. That is, advisors are suggesting that lower fertilizer rates can be used in conjunction with application processes that target better a) timing, or b) placement (broadcast vs. deep), or c) placement relative to the plant. The implication is that, at currently high prices, the reduced cost associated with the reduced rate will more than offset the added cost associated with different or additional application methods. But, what about the manager who already is applying fertilizer using the most efficient methods? Should he change his rates in the face of today's high fertilizer prices? To answer that, we still have to come back to the expected yield response function. More to the point, just because it may not have been particularly costly to ignore price impacts on optimal fertilizer rates in the past, that may not be true today? Certainly, the variation in fertilizer prices observed of late is much greater than that observed over much of the past, when many price-ignoring rate-recommendation formulas were first established.

To drive the above points home, a personal example is noteworthy. In 2001, Kastens, Schmidt, and Dhuyvetter (KSD) developed a Mitscherlich-like asymptotic plateau yield model consistent with KSU fertilizer recommendations (Kastens, et al.). Though used on the Kastens farm for several years, the KSD model's shortcomings became apparent in the face of sharply higher fertilizer prices in early 2005. In particular, the KSD corn model recommended sharply reduced N rates (around a third of KSU's recommended rates), lower than what "seemed" reasonable. Two conclusions arose on the Kastens farm. First, fertilizer rate recommendations should depend on price. Second, though a price-including model for determining optimal fertilizer rates might seem appropriate for years, it may be revealed to be erroneous in the presence of extreme fertilizer prices. Moreover, given that Dhuyvetter (at www.agmanager.info) is predicting Spring 2006 N prices that are higher yet, it is likely that incorporating price into *fertN* recommendations will be even more important for 2006 N decisions. The balance of this paper analyzes this problem in the light of KSU's current *Nrec*'s, which do not incorporate a price component.

University and commercial soil testing laboratories, as well as some crop consulting firms, routinely provide (often online) mathematical formulas depicting recommended fertilizer rates for crops. Typically, but not always, recommended rates depend on soil tests. For example, Eq. [1] shows KSU's wheat *Nrec* as described in its MF-2586 publication.

$$\begin{aligned} Wheat\ Nrec = &(2.4 * YG) - (10 * OM) - Profile\ N - Manure - Other\ N\ Adjustments \\ &+ Previous\ Crop,\ tillage,\ and\ grazing\ adjustments. \end{aligned} \qquad [1]$$

In Eq. [1], *Wheat Nrec* is recommended fertilizer N in lb N/acre, *YG* is yield goal in bu/acre, *OM* is percent soil organic matter in the top 6 inches of soil, *Profile N* is lb of nitrate nitrogen (NO_3-N) per acre in a 2-foot soil profile, often referred to more simply as lb N/acre, and the other categories are to remind the user that other N credits may need to be considered. Though rarely made explicit, *YG* typically is taken to be 110% of statistically expected (i.e., historical, or possibly trend-adjusted average) yield. The *OM* term in Eq. [1] represents the expected mineralization of organic matter to usable N fertility during the production cycle. Thus, a soil with 2.0% *OM* is expected to need 10 lb/acre less *fertN* than a soil with only 1.0% *OM*. Although not shown, since corn production occurs during warmer temperatures than wheat, implying more N mineralization, KSU's corn *Nrec* gives a credit of 20 lb N/acre for each percent *OM*; likewise for grain sorghum and sunflowers. Also, corn's *YG* factor is 1.6 rather than the 2.4 shown for wheat. Other crop *Nrec*'s have their own equations.

As discussed, to learn whether and how much price might impact optimal *fertN* rates, it is crucial that we posit a reasonable yield model underlying formula-based *Nrec*'s like those of Eq. [1]. An important component of such *Nrec*'s is *YG* and hence yield models consistent with such *Nrec*'s must themselves have a conceptual and mathematical tie-in to *YG*. As used here, we assume that *YG* conceptually is the maximum yield within an expectation framework. In Fig. 1, it is the highest point of the heavy line. As such, it also is the optimal yield if *fertN* were costless. Mathematically, in the yield models we consider, *YG* is the maximum possible yield (e.g., at infinite N levels). From a crop yield response standpoint, Eq. [1] makes it clear that *Profile N* (soil test N, or *STN*) is expected to trade off 1 for 1 with *fertN* and 10 for 1 with *OM*. This suggests that the x-axis in a yield response figure should reflect total usable N (*TUN*), and which is comprised of *fertN*+*STN*+10**OM* in the case of wheat.

A problem with devising yield models that are consistent with *Nrec* models is that an infinite number exist, even when only plateau-type functions are considered. Ultimately, the decision is an empirical one as much as it is a conceptual one. One simple model to consider is the linear plateau model itself. Fig. 2 shows such models consistent with KSU's wheat *Nrec*. Each circled knot in the figure is vertically above the x-axis value equating to KSU's *Nrec* plus the *OM* component plus *STN*. The slope of an imaginary line connecting the knots in the figure is 1/2.4 (i.e., 0.417) since 2.4 is the *YG* factor in KSU's wheat *Nrec*. Because a positive y-intercept was assumed in the figure, the slope of any line in the figure left of its knot (the responsive part) is less than 0.417 and in fact 0.25. This implies that, as long as the *fertN*-to-wheat price ratio is less than 0.25 it will be optimal to apply *fertN* until the plateau is reached. In this example figure we assumed the y-intercept to be 40% of *YG* merely as a convenience to visually distinguish the lines. More realistically, a y-intercept of 0 probably would be more appropriate given that our x-axis depicts *TUN* and not *fertN*. Moreover, because expected response can be curvilinear, the linear plateau depiction of yield in the figure is only appropriate

[1]

if it can be supported empirically. As a reminder, if linear plateau functions like those in Fig. 2 are "correct," then profit generally will not be increased by reducing *fertN* rates, even when N prices are fairly high. Of course, with high N prices it might still be appropriate to consider alternative fertilizer application methods as already discussed.

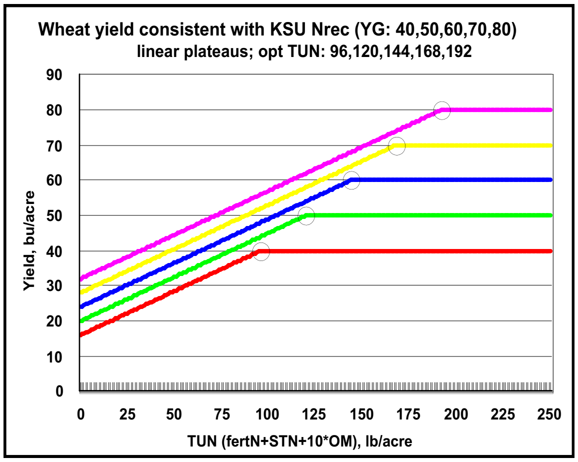

Figure 2

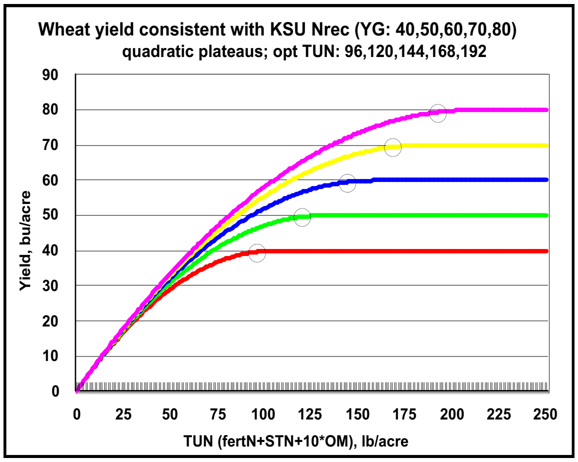

Figure 3

If the developers of KSU's *Nrec* formulas had in mind underlying curvilinear response, then they also had to have in mind an expected *fertN*-to-crop price ratio. For our purposes we assume a wheat price of $3.20/bu, which is the government-loan-adjusted average harvest (July) price in Kansas during the decade preceding KSU's *Nrec* publication (1993-2002). Similarly, the corn price (October) is $2.35. We assumed that 50% of *fertN* comes from NH3 (82% N), 25% from urea (45% N), and 25% from UAN (32% N). Based on annual USDA-NASS product prices, this results in a 1993-2002 average *fertN* price of $0.21/lb of N. Hence, we assumed *fertN*-to-crop price ratios underlying KSU *Nrec*'s to be 0.0656 (i.e., 0.21/3.20) and 0.0894 (i.e., 0.21/2.35) for wheat and corn, respectively.

Given the assumed 0.0656 *fertN*-to-wheat price ratio, Fig. 3 shows curvilinear yield models consistent with KSU's wheat *Nrec*'s at various yield goals (*YG*). The mathematical functions are 0-y-intercept quadratic plateaus where the plateaus occur at the functional peaks, which are assumed to be *YG* levels. Because each of Fig. 2 and 3 maps economic optimal levels of *fertN* to KSU *Nrec*'s, the circles in the two figures are identical from an x-axis perspective. However, with the curvilinear response of Fig. 3, each circle maps to a point whose slope exactly equals 0.0656. Thus, unlike in Fig. 2, where *Nrec* rates imply optimal yields equal to plateau yields, here *Nrec* rates imply optimal yields slightly less than plateau yields (99.26% of plateaus on average).

The next step in this work is to determine which type of mathematical function best depicts reality. External validation of this decision likely will come not only from assessing which mathematical function best fits fertilizer response data, but also from examination of the model-implied change in yield associated with a change in N rate. Consequently, we examined several functional forms, each with different degrees of curvature in the relevant areas near the plateaus. We restricted our examination to only plateau-type functions. Also, we assumed that curvilinear functions were continuous (yield is not "chopped off" at the plateau but approaches it smoothly). Below are the mathematical functions considered.

linear plateau:

$$Y = min(B_1, B_2 + B_3 * TUN). \qquad [2]$$

KSD (Mitscherlich):

$$Y = B_1 * \left(1 - B_2 * e^{-B_3 * TUN}\right); 0 \leq B_2 \leq 1. \qquad [3]$$

quadratic plateau:

$$Y = B_1 + B_2 * funcN - B_3 * funcN^2; funcN = min\left(\frac{B_2}{2 * B_3}, TUN\right). \qquad [4]$$

cubic plateau:

$$Y = B_1 + B_2 * funcN - B_3 * funcN^2 + B_4 funcN^3; funcN = min\left(\frac{B_3}{3 * B_4}, TUN\right); B_4 = \frac{B_3^2}{3 * B_2}. \qquad [5]$$

hyperbolic tangent (tanh):

$$Y = B_1 * tanh(Z); Z = G + B_3 * TUN; tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}; G = 0.5 * ln\left(\frac{1 + B_2}{1 - B_2}\right). \qquad [6]$$

In the functions specified above, B_1, B_2, and B_3 are positive numerical constants. When used to examine which functions best fit the data they are parameters to be estimated. When used to imply yield models from KSU *Nrec*'s they are calculated from stated *Nrec*'s and the assumed *fertN*-to-crop price ratio. That is, parameters are selected mathematically so that solving for the *fertN* level that equates the marginal value of *fertN* to the *fertN*-to-crop price ratio gives exactly KSU's *Nrec*. Put another way, solving the equation $\partial Y / \partial fertN = 0.0656$ for *fertN* will give a value exactly equal to the number computed from KSU's wheat *Nrec* in Eq. [1]. In the models above, B_1 is the bu/acre 0-N y-intercept for the linear, quadratic, and cubic plateaus. B_2 is the 0-N y-intercept for the KSD and tanh functions, expressed as a proportion of *YG* subtracted from *YG* for the KSD and simply the proportion of *YG* for tanh. When used to represent yield models underlying KSU *Nrec*'s, y-intercepts are assumed to be 0. However, because *STN* and *OM* typically are unknown in fertilizer trials, we allow y-intercepts to be positive in model estimation. Note that the cubic plateau is a special function that plateaus at the first inflection point, which is constrained to occur at a slope of 0 (see Beattie, et al.).

RESULTS

As described above, each of the mathematical functions can be used to exactly give back KSU *Nrec*'s at the assumed *fertN*-to-crop price ratios. Once parameters are selected, optimal *fertN* rates can be computed with other price ratios. That means mathematical formulas can be developed to adjust current KSU *Nrec* formulas taking into account *fertN* or crop price. Given a functional form, the lb/acre adjustment to optimal *fertN* rates depends on *YG* and how much price changes from the base. Of course, the percentage change in optimal *fertN* rates will depend on levels of *STN* and *OM*. That is, a given lb/a reduction in optimal *fertN* will be a greater percentage reduction where lower *fertN* rates are required in the first place (e.g., due to high levels of *STN* or *OM*). Though we later provide an exact formula for the adjustments, it may be more useful to readers to see some typical adjustments given a reasonable change in *fertN* price. In particular, we consider an increase in *fertN* price from the $0.21/lb assumed underlying KSU's *Nrec*'s to a late-2005 price of $0.40/lb (no change in crop price). Percentage changes are reported based on a base of 0 lb/acre *STN* and 2% *OM*. Tables 1 and 2 report results for wheat and corn, respectively. The tables show considerable variation among the curvilinear functions in terms of the impact on optimal *fertN* rates wrought by higher *fertN* price. The KSD function showed the greatest decline in rates and the quadratic plateau had the smallest drop in rates. Recall that the KSD model was the one which invoked questions on the Kastens farm.

Table 1. Impact on KSU Wheat *Nrec* of a change in *fertN* price from $0.21/lb to $0.40/lb for different yield models.

	Yield goal (*YG*), bu/acre				
	40	50	60	70	80
KSU *Nrec*, lb/acre	76	100	124	148	172
	Change in *Nrec* at $0.40/lb N, lb/acre				
linear plateau	0	0	0	0	0
KSD (Mitscherlich)	-21	-26	-32	-37	-42
quadratic plateau	-8	-10	-12	-14	-16
cubic plateau	-13	-17	-20	-23	-27
hyperbolic tangent (tanh)	-17	-21	-26	-30	-34
	Change in *Nrec* at $0.40/lb N, percent, if *STN* is 0 lb/acre and *OM* is 2%				
linear plateau	0%	0%	0%	0%	0%
KSD (Mitscherlich)	-28%	-26%	-26%	-25%	-25%
quadratic plateau	-11%	-10%	-10%	-10%	-10%
cubic plateau	-18%	-17%	-16%	-16%	-16%
hyperbolic tangent (tanh)	-22%	-21%	-21%	-20%	-20%

To select the most appropriate functional form for adjusting KSU *Nrec*'s we examined how well the various functions fit the data from several fertilizer N research trials across Kansas. That is, for a given research project, we first fit response data from each site-year to a linear plateau function. When a linear function did not plateau we assumed the plateau to be that associated with the highest *fertN* application rate in the trial. Then, we averaged the linear plateau values to generate a collection of linear segments like that represented by the heavy line in Fig. 1. Finally, we fit the various functions to this "heavy line" for each research project. We should note that our rule for non-plateauing site-years likely biased our results slightly in favor of the linear plateau as the expected generalizing response.

Tribune Kansas data were provided by Alan Schlegel, with wheat data from cooperator plots and corn and grain sorghum data from the Research Station. Yield response for Tribune wheat was analyzed both for *fertN* and for *TOTN*, which was comprised of *fertN* + *STN*. Treatments were 0, 20, 40, 60, 80, and 100 lb/acre *fertN*. Tribune irrigated corn and grain sorghum treatments included 0, 40, 80, 120, 160, and 200 lb/acre. Republic County Kansas grain sorghum data were provided by Barney Gordon, agronomist-in-charge of KSU's North

Table 2. Impact on KSU Corn *Nrec* of a change in *fertN* price from \$0.21/lb to \$0.40/lb for different yield models.

	Yield goal (*YG*), bu/acre				
	70	90	110	130	150
KSU *Nrec*, lb/acre	72	104	136	168	200
	Change in *Nrec* at \$0.40/lb N, lb/acre				
linear plateau	0	0	0	0	0
KSD (Mitscherlich)	-24	-30	-37	-44	-50
quadratic plateau	-9	-11	-13	-16	-18
cubic plateau	-14	-18	-23	-27	-31
hyperbolic tangent (tanh)	-19	-25	-30	-35	-41
	Change in *Nrec* at \$0.40/lb N, percent, if *STN* is 0 lb/acre and *OM* is 2%				
linear plateau	0%	0%	0%	0%	0%
KSD (Mitscherlich)	-33%	-29%	-27%	-26%	-25%
quadratic plateau	-12%	-11%	-10%	-9%	-9%
cubic plateau	-20%	-18%	-17%	-16%	-15%
hyperbolic tangent (tanh)	-27%	-24%	-22%	-21%	-20%

Central Kansas Experiment Fields. Prior to 1996, treatments were 0, 30, 60, and 90 lb/acre, whereupon it became apparent that 90 lb/acre was insufficient to cause yields to plateau. Hence, beginning in 1996, treatments of 120, 150, 180, and 210 lb/acre were added to the experiment. Consequently, we examine both the entire time series, as well as the later period by itself. Garden City 1991-1994 irrigated corn and grain sorghum data originated from research by Jim Schaffer, head of that Research Station at the time, along with Schlegel and Dhuyvetter.

Table 3 shows the root mean squared error (RMSE) for each of the models in the various data sets. In the table it can be seen that, among the functions considered, the quadratic plateau function most frequently was best, had the lowest RMSE on average, and was ranked the best on average. Conversely, the linear plateau was either worst or second-worst for these same categories. Hence, it is reasonable to conclude that: 1) expected *fertN* response generally is curvilinear, meaning that price matters to optimal fertilizer rates; and 2) if we were to select one functional form to generate adjustments to KSU *Nrec*'s it likely should be the quadratic plateau.

To gain some insight into the subtleties involved in the model fits, Fig. 4 shows the "heavy line" being fit and the various model estimates for grain sorghum rotated with soybeans in Republic County from 1996-2002. Though the quadratic plateau is best fitting in this case, differences among models are not great. Yet, as we know from tables 1-2, subtle differences in models can result in significant changes in optimal fertilizer rates in the face of changing *fertN* prices.

As another graphical example of interest, Fig. 5 shows response to *TOTN* for fall injected *fertN* in Tribune wheat. The line being fit and the various curvilinear functions are nearly on top of each other, revealing substantial curvature and in sharp contrast to the linear plateau as a generalizing function. Though not shown, this response to *TOTN* is more curved, especially at low N levels, than the comparable response to *fertN*. This provides additional support for using curvilinear functions for adjusting KSU *Nrec*'s when assuming response is to *TUN*, not *fertN*.

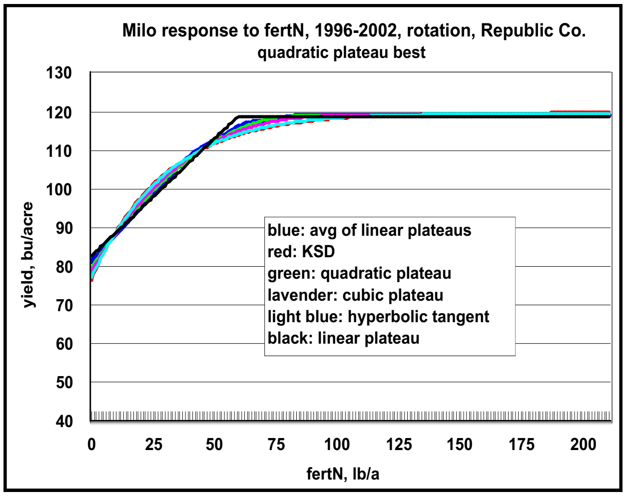

Figure 4

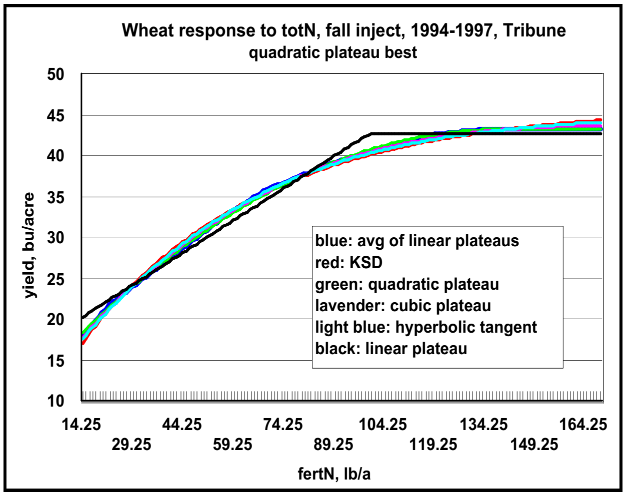

Figure 5

Table 3. Root mean squared error (RMSE) for models fitted to linear plateau response values from N research trials.

			Yield model				
description	response to:	time	linear plateau	KSD	quadratic plateau	cubic plateau	tanh
Tribune wheat, fall inject	*fertN*	1994-1997	**0.2556**[a]	0.3539	0.2825	0.3205	0.3275
Tribune wheat, winter broadcast	*fertN*	1994-1997	0.5027	**0.0962**	0.1502	0.1206	0.1093
Tribune wheat, spring inject	*fertN*	1994-1997	0.3621	0.3964	**0.1773**	0.2782	0.3574
Tribune wheat, spring broadcast	*fertN*	1994-1997	0.1349	0.1335	**0.1241**	0.1289	0.1267
Tribune wheat, fall inject	*TOTN*	1994-1997	0.9348	0.5070	**0.1917**	0.2571	0.3749
Tribune wheat, winter broadcast	*TOTN*	1994-1997	0.9912	**0.1745**	0.3555	0.2422	0.2148
Tribune wheat, spring inject	*TOTN*	1994-1997	0.8284	0.2716	0.2727	**0.1183**	0.1978
Tribune wheat, spring broadcast	*TOTN*	1994-1997	1.3739	**0.2138**	0.7334	0.4897	0.3475
Tribune irrigated sorghum	*fertN*	1961-2004	0.6608	1.5366	**0.5099**	0.8279	1.3580
Tribune irrigated corn	*fertN*	1961-2005	1.7466	2.7305	**0.8690**	1.6259	2.2568
Republic Co. sorghum, continuous	*fertN*	1982-2002	0.8894	1.3501	**0.5366**	0.7752	1.1276
Republic Co. sorghum, in rotation	*fertN*	1982-2002	0.9854	0.5665	0.7010	0.6196	**0.5375**
Republic Co. sorghum, continuous	*fertN*	1996-2002	**0.5561**	2.4951	1.0373	1.5683	2.1609
Republic Co. sorghum, in rotation	*fertN*	1996-2002	0.7346	1.3190	**0.3562**	0.6466	1.1714
Garden City irr. corn, after corn	*fertN*	1991-1994	2.5005	2.3974	**1.2505**	1.7347	2.1289
Garden City irr. corn, after soybeans	*fertN*	1991-1994	0.9219	0.3340	0.5000	0.3405	**0.3225**
Garden City irr. sorg., after sorg.	*fertN*	1991-1994	1.3601	1.1240	**0.7284**	0.7874	1.0258
Garden City irr. sorg., after soybeans	*fertN*	1991-1994	**0.0235**	0.2056	0.0910	0.1280	0.2035
average RMSE			0.8757	0.9003	0.4926	0.6116	0.7972
average rank (1 best ... 5 worst)			3.78	3.67	2.17	2.50	2.89
number of times model is best			3	3	9	1	2

[a] bold font indicates best fitting model for a row

Adjusting KSU Nrec's to accommodate price can be cast in a framework of a base fertN-to-crop price ratio, which we refer to as R_{base}, and a current or expected price ratio, which we refer to simply as R. As discussed earlier, in our analysis we used an R_{base} of 0.0656 and 0.0894 for wheat and corn, respectively (from these historical prices: wheat, \$3.20/bu; corn, \$2.35/bu; fertN, \$0.21/lb). We used a 0-y-intercept quadratic plateau in our calculations. But, in terms of recommended adjustments to KSU Nrec's, the process is not particularly sensitive to either R_{base} or whether we had used a positive y-intercept. Without showing the supporting mathematics, we merely report the following recommended lb N/acre additive adjustment to a KSU Nrec, where YG_{fac} is the factor multiplied by the YG in the Nrec formula (e.g., 2.4 for wheat and 1.6 for corn):

$$additive\,adjustment = \frac{YG * YG_{fac}^{2}}{1+\left(1-2*YG_{fac}*R_{base}\right)^{0.5}-YG_{fac}*R_{base}}*\left(R_{base}-R\right). \qquad [7]$$

Though tedious, Eq. [7] is easy to program in a spreadsheet, making adoption straightforward. Where desired, application and rotational adjustments should be made first, followed next by the formal adjustments specified in Eq. [7].

A side benefit to the process developed in this work is that reasonable yield models naturally arise from the analysis. For example, if we assume that the quadratic plateau function of Eq. [4] is the selected form, along with the assumption of a 0-y-intercept (i.e., $B_1 = 0$), B_2 and B_3 can be calculated as follows:

$$B_2 = \frac{1+\left(1-2*YG_{fac}*R_{base}\right)^{0.5}}{YG_{fac}}. \qquad [8]$$

$$B_3 = \frac{B_2 - R_{base}}{2*YG*YG_{fac}}. \qquad [9]$$

Now, a reasonable quadratic plateau function is completely specified and it can be used for making other N decisions, for example, an analysis of benefits associated with soil testing or an analysis of site-specific N treatment.

SUMMARY

Building upon a discussion of linear and curvilinear yield response to fertilizer, this research developed a framework where response is fundamentally linear for any particular site-year, but where expected response can become curvilinear in the face of random weather across space and time. We next put forth several functional forms as potential candidates for generalizing *expected* yield response to N. The functional forms were evaluated using historical N trial data from western and north central Kansas involving wheat, corn, and grain sorghum. The quadratic plateau functional form arose as the candidate of choice from that analysis. Given the doubling of fertilizer N prices in the last two years, that functional form implies N cutbacks around 10 lb/acre for 50 bu/acre wheat to around 30 lb/acre for 250 bu/acre corn. Finally, we presented the mathematics required to compute our suggested adjustments to current KSU N recommendations to accommodate changes in N prices.

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